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Short Communication

A prediction method for aerodynamic sound produced by multiple elements in air ducts

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Abstract

A prediction method for aerodynamic sound produced by the interaction of multiple elements in a low speed flow duct has been developed. Same as the previous works of Mak and Yang for two in-duct elements, the concept of partially coherent sound fields is adopted to formulate the sound powers produced by interaction of multiple in-duct elements at frequencies below and above the cut-on frequency of the lowest transverse duct mode. An interaction factor is finally defined as a result of a simple relationship between the sound power due to the interaction of multiple in-duct elements and that due to a single in-duct element. The present study suggests that it is possible to predict the level and spectral distribution of the additional acoustic energy produced by the interaction of multiple in-duct elements. The proposed method therefore can form a basis of a generalized prediction method for aerodynamic sound produced by multiple in-duct elements in a ventilation system.

1. Introduction

Being able to accurately predict aerodynamic sound produced by elements in air ducts at the design stage is important in engineering, since duct discontinuities such as dampers and bends are unavoidable, and it is almost impossible to remedy the problems after the ventilation system has

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been installed. Current design guides such as CIBSE guide [1] and ASHRAE handbook [2] have provided the design method for predicting the aerodynamic sound produced by an isolated in-duct element in the ventilation system. However, these guides are based on the experimental work of a number of investigators who worked on a limited range of in-duct components and a limited range of duct sizes. In addition, they underestimated the levels of aerodynamic sound production in practical systems. Over the years, a number of investigators [3–10] have tried to devise a generalized method of predicting the aerodynamic sound production in a low speed flow duct. Nevertheless, most predictive methods are only applied to an isolated in-duct element that is very different from that found in practical systems. Recent works of Mak and Yang [11,12] have suggested that it is possible to predict the flow-generated noise produced by two in-duct strip spoilers. Their predicted results have been compared to the experimental results of Ukpoho and Oldham [13]. However, their technique is only limited to two in-duct elements. The specific problem that motivated this study is that there are always multiple in-duct elements (more than two elements) in a ventilation system. The aim of the present investigation is therefore to predict sound level and spectral content of the aerodynamic sound produced by multiple elements in low speed air flow ducts.

2. Sound power radiated by multiple flow spoilers in air duct

2.1. Interaction of multiple noise sources

At low Mach numbers, the problem of sound produced by a flow spoiler in a duct can be treated by replacing the spoiler and the turbulence by a distribution of dipole sources radiating into a duct filled with fluid at rest. The spectral density $W(\omega)$ of the acoustic power, radiated in one direction down an infinite rectangular duct can be obtained from Ref. [7] as follows:

$$W(\omega) = \sum_{m,n}^{N} \frac{\rho_0 c_{mn}}{4A} S_{mn}(\omega), \tag{1}$$

where N represents the upper limit of the summation (restricted to propagating modes), ρ_0 is the ambient air density, c_{mn} is the mode axial phase speed and A is the duct area of cross-section. The quantity $S_{mn}(\omega)$ represents the power spectral density of the source volume integral $Q_{mn}(\omega)$, as defined by

$$S_{mn}(\omega) = \lim_{T\to\infty} \frac{\pi}{T} |Q_{mn}(\omega)|^2. \tag{2}$$

When there are M ($M > 2$) number of sound sources in the air duct, the distance between ith sound source and $(i+1)$th sound source and that between $(i-1)$th sound source and $(i+j)$th sound source are represented by $d_{i(i+1)}$ and $d_{(i-1)(i+j)}$, respectively, M, i and j are integers and $j = 1, 2, \ldots, (M-2)$. The total power spectral density of the acoustic power radiated by M number of sound sources in one direction down an infinite rectangular duct is

$$W(\omega) = \sum_{mn} \frac{1}{4A} \rho_0 c_{mn} \Bigg\{ \sum_{i=1}^{M} S_{mn}^{ii}(\omega) + \sum_{i=1}^{M-1} \Big\{ \sqrt{\gamma_{i(i+1)}^2} [S_{mn}^{i(i+1)}(\omega) + S_{mn}^{(i+1)i}(\omega)] \cos(k_{mn} d_{i(i+1)}) \Big\} + \sum_{i=1}^{M-1} \Big\{ \sqrt{\gamma_{(i-1)(i+j)}^2} [S_{mn}^{(i-1)(i+j)}(\omega) + S_{mn}^{(i+j)(i-1)}(\omega)] \cos(k_{mn} d_{(i-1)(i+j)}) \Big\} \Bigg\}, \tag{3}$$

where all quantities $S_{mn}(\omega)$ represent the power spectral density of the corresponding source volume integral $Q_{mn}(\omega)$, as defined by

$$S_{mn}^{ii}(\omega) = \lim_{T\to\infty} \frac{\pi}{T} |Q_{mn}^{i}(\omega)|^2, \tag{4}$$

$$S_{mn}^{i(i+1)}(\omega) = S_{mn}^{(i+1)i^*}(\omega) = \lim_{T\to\infty} \frac{\pi}{T} (Q_{mn}^{i} Q_{mn}^{(i+1)^*}), \tag{5}$$

$$S_{mn}^{(i-1)(i+j)}(\omega) = S_{mn}^{(i+j)(i-1)^*}(\omega) = \lim_{T\to\infty} \frac{\pi}{T} (Q_{mn}^{(i-1)} Q_{mn}^{(i+j)^*}), \tag{6}$$

for $j = 1, \ldots, (M-2), M > 2$ and value of $S_{mn}^{xy}(\omega)$ is ignored if any one of its superscripts x or y is zero or greater than M. $\gamma_{i(i+1)}^2$ and $\gamma_{(i-1)(i+j)}^2$ are the coherence function between ith sound source and $(i+1)$th sound source and the coherence function between $(i-1)$th sound source and $(i+j)$th sound source, respectively.

2.2. *The discussion of solution for sound produced by multiple in-duct flow spoilers in an infinite hard walled duct*

Further progress towards a simple approximation for $S_{mn}(\omega)$ can be made by relating it to the power spectrum $S_F(\omega)$ of the total drag force on the spoiler. The resulting expression of Nelson and Morfey [7] for the spectral density $S_{mn}(\omega)$ is

$$S_{mn}(\omega) = \frac{1}{\rho_0^2 |c_{mn}|^2} S_F(\omega) \frac{1}{A_s} \int_{A_s} \int |\psi_{mn}(x_k)|^2 \, \mathrm{d}s(x_k). \tag{7}$$

In addition, for non-zero (m, n),

$$\frac{1}{A_s} \int_{A_s} \int |\psi_{mn}(x_k)|^2 \, \mathrm{d}s(x_k) = \left[1 + \frac{a}{m\pi l} \sin\left(\frac{m\pi l}{a}\right) \cos\left(\frac{2m\pi \bar{a}}{a}\right)\right] \times \left[1 + \frac{b}{m\pi r} \sin\left(\frac{m\pi r}{b}\right) \cos\left(\frac{2m\pi \bar{b}}{b}\right)\right] \approx 1, \tag{8}$$

where $(\bar{a}, \bar{b})$ is the spoiler central coordinate, a, b are the duct cross-section dimensions, and l, r are length and width of the spoiler, respectively. Similarly, Eqs. (4)–(6) can be written as

$$S_{mn}^{ii}(\omega) = \frac{1}{\rho_0^2 |c_{mn}|^2} S_{Fi}(\omega), \tag{9}$$

$$S_{mn}^{i(i+1)}(\omega) = S_{mn}^{(i+1)i}(\omega) = \sqrt{S_{mn}^{ii}S_{mn}^{(i+1)(i+1)}}\cos[\phi_{i(i+1)}(\omega)] = \frac{1}{\rho_0^2|c_{mn}|^2}\sqrt{S_{Fi}(\omega)S_{F(i+1)}(\omega)}\cos[\phi_{i(i+1)}(\omega)], \quad (10)$$

$$S_{mn}^{(i-1)(i+j)}(\omega) = S_{mn}^{(i+j)(i-1)}(\omega) = \sqrt{S_{mn}^{(i-1)(i-1)}S_{mn}^{(i+j)(i+j)}}\cos[\phi_{(i-1)(i+j)}(\omega)] = \frac{1}{\rho_0^2|c_{mn}|^2}\sqrt{S_{F(i-1)}(\omega)S_{F(i+j)}(\omega)}\cos[\phi_{(i-1)(i+j)}(\omega)], \quad (11)$$

where $S_{Fi}(\omega)$, $S_{F(i+1)}(\omega)$, $S_{F(i-1)}(\omega)$ and $S_{F(i+j)}(\omega)$ are the power spectrum of the total fluctuating drag forces acting on the ith, $(i+1)$th, $(i-1)$th and $(i+j)$th spoilers, respectively. In order to have a more convenient analysis, it is defined that $S_{mn}^{i(i+1)} \equiv |S_{mn}^{(i+1)i}|e^{i\phi_{i(i+1)}(\omega)}$ in Eq. (10) and $S_{mn}^{(i-1)(i+j)} \equiv |S_{mn}^{(i-1)(i+j)}|e^{i\phi_{(i-1)(i+j)}(\omega)}$ in Eq. (11), where $\phi_{i(i+1)}(\omega)$ and $\phi_{(i-1)(i+j)}(\omega)$ represent the phase of the cross-power spectral density $S_{F_iF_{(i+1)}}$ and $S_{F_{(i-1)}F_{(i+j)}}$, respectively.

Therefore, Eq. (3) for the spectral density of the radiated sound power has the following form:

$$W(\omega) = \frac{1}{4A\rho_0}\sum_{m,n}^{N}\frac{1}{|c_{mn}|}\left[\sum_{i=1}^{M}S_{F_i}(\omega) + \sum_{i=1}^{M-1}\left\{\sqrt{\gamma_{i(i+1)}^2}\,2\cos[\phi_{i(i+1)}(\omega)]\cos(k_{mn}d_{i(i+1)})\sqrt{S_{F_i}(\omega)S_{F_{(i+1)}}(\omega)} + \sqrt{\gamma_{(i-1)(i+j)}^2}\,2\cos[\phi_{(i-1)(i+j)}(\omega)]\cos(k_{mn}d_{(i-1)(i+j)})\sqrt{S_{F_{(i-1)}}(\omega)S_{F_{(i+j)}}(\omega)}\right\}\right]. \quad (12)$$

For plane wave propagation, i.e. $m, n = 0$, at all frequencies below the cut-on frequency of the first transverse duct mode, Eq. (12) reduces to

$$W(\omega) = \frac{1}{4A\rho_0c_0}\left[\sum_{i=1}^{M}S_{F_i}(\omega) + \sum_{i=1}^{M-1}\left\{\sqrt{\gamma_{i(i+1)}^2}\,2\cos[\phi_{i(i+1)}(\omega)]\cos(kd_{i(i+1)})\sqrt{S_{F_i}(\omega)S_{F_{(i+1)}}(\omega)} + \sqrt{\gamma_{(i-1)(i+j)}^2}\,2\cos[\phi_{(i-1)(i+j)}(\omega)]\cos(kd_{(i-1)(i+j)})\sqrt{S_{F_{(i-1)}}(\omega)S_{F_{(i+j)}}(\omega)}\right\}\right]. \quad (13)$$

In the case of multimodal sound propagation in the duct, the summation of $(1/c_{mn})$ and that of $[1/c_{mn}\cos(k_{mn}d)]$ for all the propagating modes should be solved so that Eq. (12) can be applied for practical engineering need. First, it can be noted that the ratios (c_0/c_{mn}) for a given mode can be expressed in terms of the integral m and n, i.e.

$$c_0/c_{mn} = \sqrt{1 - (m\pi/ka)^2 - (n\pi/kb)^2}.$$

If m and n are now regarded as continuous variables, the ratio (c_0/c_{mn}) can be thought of as another continuous variable, which is a function of m and n, i.e. $(c_0/c_{mn}) \approx f(m,n)$. Thus one can estimate

$$\sum_{m,n}^{N}\frac{1}{c_{mn}} = \frac{1}{c_0}\sum_{m,n}^{N}\frac{c_0}{c_{mn}} \approx \frac{1}{c_0}\int_0^{kb/\pi}\int_0^{ka/\pi} f(m,n)\,\mathrm{d}m\,\mathrm{d}n, \quad (14)$$

where (ka/π) and (kb/π) are the maximum values of the continuous variables m and n for which modes propagate at the frequency in question. According to the concept in Ref. [7], the summation is given by

$$\sum_{m,n}^{N} \frac{c_0}{c_{mn}} \approx \frac{k^2 ab}{6\pi} + \frac{k}{8}(a+b). \tag{15}$$

Similarly, the sum of $\sum_{m,n}^{N}(c_0/c_{mn})\cos(k_{mn}d)$ can be derived as follows:

$$\begin{aligned}\sum_{m,n}^{N}\left(\frac{c_0}{c_{mn}}\right)\cos(k_{mn}d) &\approx \int\int_s \sqrt{1-\left(\frac{m\pi}{ka}\right)^2-\left(\frac{n\pi}{kb}\right)^2}\cos\left[(kd)\sqrt{1-\left(\frac{m\pi}{ka}\right)^2-\left(\frac{n\pi}{kb}\right)^2}\right]\mathrm{d}m\,\mathrm{d}n \\ &= \frac{k^2ab}{2\pi}\left[\frac{\sin e}{e}+\frac{2\cos e}{e^2}-\frac{2\sin e}{e^3}\right]+\frac{k(a+b)}{4}\left[J_0(e)-\frac{J_1(e)}{e}\right],\end{aligned} \tag{16}$$

where $e = kd$, d is the distance between two spoilers and J_0 and J_1 are the zero- and first-order Bessel's functions, respectively. The calculation of this summation is given in Appendix A of Ref. [11]. Substitution of this approximation for the summation in Eq. (12) then gives the expression for the spectral density of the sound power radiated down in the duct due to the two spoilers:

$$\begin{aligned}W(\omega) = \frac{\omega^2}{24\pi\rho_0 c_0^3}\left[1+\frac{3\pi c_0}{4\pi}\frac{(a+b)}{A}\right]\Bigg[\sum_{i=1}^{M} S_{F_i}(\omega) \\ + \sum_{i=1}^{M-1}\Big\{2Q_{i(i+1)}\sqrt{\gamma^2_{i(i+1)}}\cos[\phi_{i(i+1)}(\omega)]\sqrt{S_{F_i}(\omega)S_{F_{(i+1)}}(\omega)} \\ + 2Q_{(i-1)(i+j)}\sqrt{\gamma^2_{(i-1)(i+j)}}\cos[\phi_{(i-1)(i+j)}(\omega)]\sqrt{S_{F_{(i-1)}}(\omega)S_{F_{(i+j)}}(\omega)}\Big\}\Bigg],\end{aligned} \tag{17}$$

where $Q_{i(i+1)}$ is given by

$$Q_{i(i+1)} = \frac{\frac{k^2ab}{6\pi}3\left[\frac{\sin e}{e}+\frac{2\cos e}{e^2}-\frac{2\sin e}{e^3}\right]+\frac{k(a+b)}{8}2\left[J_0(e)-\frac{J_1(e)}{e}\right]}{\left[\frac{k^2ab}{6\pi}+\frac{k(a+b)}{8}\right]} \quad \text{and} \quad e = kd_{i(i+1)} \tag{18}$$

and $Q_{(i-1)(i+j)}$ is given by

$$Q_{(i-1)(i+j)} = \frac{\frac{k^2ab}{6\pi}3\left[\frac{\sin e}{e}+\frac{2\cos e}{e^2}-\frac{2\sin e}{e^3}\right]+\frac{k(a+b)}{8}2\left[J_0(e)-\frac{J_1(e)}{e}\right]}{\left[\frac{k^2ab}{6\pi}+\frac{k(a+b)}{8}\right]} \quad \text{and} \quad e = kd_{(i-1)(i+j)}. \tag{19}$$

Thus, the expression of sound power radiated by two spoiler sources at frequencies below and above cut-on are obtained in Eqs. (13) and (17) which are under the conditions of plane wave and multimodal sound propagation in an infinite duct, respectively.

3. A scaling law for practical engineering

3.1. The relationship between fluctuating and steady-state drag forces

In general, the rms fluctuating drag force acting on the ith spoiler is assumed directly proportional to the steady-state drag force $\bar{F}_{zi}$. This assumption was used by Gordon [4,5], Nelson and Morfey [7], Mak and Yang [11] and was supported by the experiment of Heller and Widnall [6]. It can be expressed as

$$(F_{zi})_{\rm rms} = K(St)\bar{F}_{zi} \quad (\text{band } f_c/\alpha \text{ to } f_c\alpha), \tag{20}$$

where the numerical factor $K(St)$ depends on the choice of α, f_c is central frequency. The Strouhal number is given by $St = f_c r/U_c$, where U_c is the flow velocity in the constriction provided by the spoiler (U_c is defined by the volume flow rate q and the area of the duct constriction A_c, such that $U_c = q/A_c$) and r is a characteristic dimension, and the mean drag force acting on the spoiler can be defined as

$$F_z \equiv C_D(\tfrac{1}{2}\rho U_\infty^2)A_s = C_D(\tfrac{1}{2}\rho U_c^2)\sigma^2(1-\sigma)A, \tag{21}$$

where C_D is the drag coefficient, $\sigma = A_c/A$ is the open area ratio and $U_\propto$ is the duct velocity q/A.

3.2. The relationship between sound power and flow parameters

By using previously derived Eqs. (13) and (17) for the sound power transmitted down the duct under plane wave and multimode radiation conditions, the sound power radiated in a given bandwidth can be estimated as follows.

The mean square value of the fluctuating force in a given band is given by

$$(\bar{F}_{zi}^2)_{\Delta f} = \int_{\omega_1}^{\omega_2} S_{F_i}(\omega)\,\mathrm{d}\omega. \tag{22}$$

Assuming the ratio of mean drag forces acting on the two spoilers is $\bar{F}_{zi} = \zeta_i \bar{F}_{z1}$ (i.e. $\zeta_1 = 1$) where ζ_i is a constant ratio of mean drag forces acting on the ith spoiler and the first spoiler, Eqs. (20)–(22) can then be used to express the sound power in terms of drag coefficient. After some algebraic manipulation it can be shown that:

For $f_c < f_0$ (plane wave propagation),

$$\begin{aligned} W_{\Delta f} &\approx (1/4A\rho_0 c_0)\Bigg\{\sum_{i=1}^{M}(\bar{F}_{z1}^2)_{\Delta f} + 2\sum_{i=1}^{M-1}\Bigg[\sqrt{\gamma_{i(i+1)}^2}\cos[\phi_{i(i+1)}(\omega_c)]\cos(\omega_c d_{i(i+1)}/c_0)\sqrt{(\bar{F}_{zi}^2)_{\Delta f}(\bar{F}_{z(i+1)}^2)_{\Delta f}} \\ &\quad + \sqrt{\gamma_{(i-1)(i+j)}^2}\cos[\phi_{(i-1)(i+j)}(\omega_c)]\cos(\omega_c d_{(i-1)(i+j)}/c_0)\sqrt{(\bar{F}_{z(i-1)}^2)_{\Delta f}(\bar{F}_{z(i+j)}^2)_{\Delta f}}\Bigg]\Bigg\} \\ &= \left(\frac{\rho_0}{16c_0}\right)AK^2(St)[\sigma^2(1-\sigma)]^2 C_D^2 U_c^4\Bigg\{\sum_{i=1}^{M}\zeta_i^2 + 2\sum_{i=1}^{M-1}\Bigg[\sqrt{\gamma_{i(i+1)}^2}\cos[\phi_{i(i+1)}(\omega_c)]\cos(\omega_c d_{i(i+1)}/c_0)\zeta_i\zeta_{i+1} \\ &\quad + \sqrt{\gamma_{(i-1)(i+j)}^2}\cos[\phi_{(i-1)(i+j)}(\omega_c)]\cos(\omega_c d_{(i-1)(i+j)}/c_0)\zeta_{i-1}\zeta_{i+j}\Bigg]\Bigg\}. \end{aligned} \tag{23}$$

For $f_c > f_0$ (multimodal propagation),

$$W_{\Delta f} \approx (\omega_c^2/24\pi\rho_0 c_0^3)[1 + (3\pi c_0/4\omega_c)(a+b)/A]\left\{\sum_{i=1}^{M}(\bar{F}_{zi}^2)_{\Delta f} + 2\sum_{i}^{M-1}\left[Q_{i(i+1)}\sqrt{\gamma_{i(i+1)}^2}\cos[\phi_{i(i+1)}(\omega_c)]\sqrt{(\bar{F}_{zi}^2)_{\Delta f}(\bar{F}_{z(i+1)}^2)_{\Delta f}} + Q_{(i-1)(i+j)}\sqrt{\gamma_{(i-1)(i+j)}^2}\cos[\phi_{(i-1)(i+j)}(\omega_c)]\sqrt{(\bar{F}_{z(i-1)}^2)_{\Delta f}(\bar{F}_{z(i+j)}^2)_{\Delta f}}\right]\right\}$$
$$= (\rho_0\pi/24c_0^3)[1 + (3\pi c_0/4\omega_c)(a+b)/A](A/r)^2(St)^2K^2(St)[\sigma^2(1-\sigma)]^2C_D^2U_c^6\left\{\sum_{i=1}^{M}\zeta_i^2 + 2\sum_{i=1}^{M-1}\left[Q_{i(i+1)}\sqrt{\gamma_{i(i+1)}^2}\cos[\phi_{i(i+1)}(\omega_c)]\zeta_i\zeta_{i+1} + Q_{(i-1)(i+j)}\sqrt{\gamma_{(i-1)(i+j)}^2}\cos[\phi_{(i-1)(i+j)}(\omega_c)]\zeta_{i-1}\zeta_{i+j}\right]\right\}, \tag{24}$$

where a, b are the duct cross-section dimensions, d is the distance between two spoilers and f_0 is the cut-on frequency of the first transverse duct mode (i.e. least non-zero value of the cut-on frequency defined by $f_0 = c_0\sqrt{(m\pi/a)^2 + (n\pi/b)^2}$ where $m, n = 0, 1, \ldots$). Since the sound power measurements are usually made in proportional frequency bands, the above scaling laws may be used as they stand to normalize the experimental data. The radiated sound power levels in $\frac{1}{3}$ octaves are thus normalized by evaluating:

For $f_c < f_0$:

$$120 + 20\log_{10}K(St) = \mathrm{SWL}_M - 10\log_{10}\{\rho_0 A[\sigma^2(1-\sigma)]^2C_D^2U_c^4/16c_0\} - 10\log_{10}\left\{\sum_{i=1}^{M}\zeta_i^2 + 2\sum_{i=1}^{M-1}\left[\sqrt{\gamma_{i(i+1)}^2}\cos(\omega_c d_{i(i+1)}/c_0)\cos[\phi_{i(i+1)}(\omega_c)]\zeta_i\zeta_{i+1} + \sqrt{\gamma_{(i-1)(i+j)}^2}\cos(\omega_c d_{(i-1)(i+j)}/c_0)\cos[\phi_{(i-1)(i+j)}(\omega_c)]\zeta_{i-1}\zeta_{i+j}\right]\right\}. \tag{25}$$

For $f_c > f_0$:

$$120 + 20\log_{10}K(St) = \mathrm{SWL}_M - 10\log_{10}\{\rho_0\pi A^2(St)^2[\sigma^2(1-\sigma)]^2[C_D^2U_c^6/24c_0^3r^2] \times [1 + (3\pi c_0/4\omega_c)(a+b)/A]\} - 10\log_{10}\left\{\sum_{i=1}^{M}\zeta_i^2 + 2\sum_{i=1}^{M-1}\left[\sqrt{\gamma_{i(i+1)}^2}Q_{i(i+1)} \times \cos[\phi_{i(i+1)}(\omega_c)]\zeta_i\zeta_{i+1} + \sqrt{\gamma_{(i-1)(i+j)}^2}Q_{(i-1)(i+j)}\cos[\phi_{(i-1)(i+j)}(\omega_c)]\zeta_{i-1}\zeta_{i+j}\right]\right\}. \tag{26}$$

Comparing the above expressions with those derived by Nelson and Morfey [7] for the sound power generated by an in-duct spoiler, the interaction factor β_M, similar to Ref. [12], can be defined as follows:

$$\beta_M = \begin{cases} \sum_{i=1}^{M} \zeta_i^2 + 2\sum_{i=1}^{M-1}\Big[\sqrt{\gamma^2_{i(i+1)}}\cos(\omega_c d_{i(i+1)}/c_0)\cos[\phi_{i(i+1)}(\omega_c)]\zeta_i\zeta_{i+1} \\ \quad +\sqrt{\gamma^2_{(i-1)(i+j)}}\cos(\omega_c d_{(i-1)(i+j)}/c_0)\cos[\phi_{(i-1)(i+j)}(\omega_c)]\zeta_{i-1}\zeta_{i+j}\Big], & f_c < f_0, \\ \sum_{i=1}^{M} \zeta_i^2 + 2\sum_{i=1}^{M-1}\Big[\sqrt{\gamma^2_{i(i+1)}}Q_{i(i+1)}\cos[\phi_{i(i+1)}(\omega_c)]\zeta_i\zeta_{i+1} \\ \quad +\sqrt{\gamma^2_{(i-1)(i+j)}}Q_{(i-1)(i+j)}\cos[\phi_{(i-1)(i+j)}(\omega_c)]\zeta_{i-1}\zeta_{i+j}\Big], & f_c > f_0, \end{cases} \quad (27a,b)$$

where $M > 2, M, i$ and j are integers and $j = 1, 2, \ldots, (M-2)$. In the above equation, it any value is ignored if any one of its subscripts is zero or greater than M.

It can be seen that β_M is dependent on the centre frequency, coherence of two spoiler sources, the distance between the two spoilers, phase of cross-power spectral density of source volumes and the ratio of mean drag forces, etc. If the sound power level due to an in-duct spoiler is denoted as SWL_s, a simple relationship between SWL_M sound power due to a multiple (M) elements and that due to a single element is then obtained as follows:

$$\mathrm{SWL}_M = \mathrm{SWL}_s + 10\log_{10}\beta_M, \quad (28)$$

where SWL_s can be obtained by using the pressure-based method provided by Nelson and Morfey [7] and β_M can be determined by numerical computation and experiments. This will require further work. The second term on the right of Eq. (28) represents the increase in flow-generated sound power level compared to the sound power level due to a multiple duct element. From Eqs. (25)–(28), it can be seen that for a known sound power level SWL_s, the overall level of aerodynamic sound SWL_M is mainly a function of the frequency and distance between the various duct elements. It increases when the interaction factor $\beta_M > 1$. It decreases when the interaction factor $0 < \beta_M < 1$. It can be seen in Eq. (27a,b) that the coherence of various duct elements and the phase of the cross-power spectral density of source volumes play an important role in the interaction of multiple in-duct elements.

It can be seen that these predictive equations (Eqs. (25) and (3.2)) are similar to those for a single element derived by Nelson and Morfey [7] and those for two elements derived by Mak and Yang [11,12]. They can be expressed in terms of measurable engineering parameters, together with the single Strouhal number dependent constant $K(St)$. The additional terms in Eqs. (25) and (3.2) as they are compared to equations of Nelson and Morfey represent the interaction between multiple aerodynamic noise sources. Nelson and Morfey measured the sound power levels radiated by different flat plate flow spoilers in a rectangular air duct at various velocities and finally they produced a normalized spectrum [7]. It should be noted that 6 dB corrections need to be subtracted from the values of $K^2(St)$ in their normalized spectrum [14]. Together with the corrected normalized spectrum $K^2(St)$ of Nelson and Morfey, the inferred-duct values of sound power level in $\frac{1}{3}$ octave bands radiated by multiple spoilers can be predicted by Eqs. (25) and (3.2).

The predictive equations (25) and (3.2) can therefore form a basis of a generalized prediction method for aerodynamic sound generated by multiple in-duct elements.

4. Conclusions

The predictive equations developed here which permit the determination of sound power radiated by multiple in-duct flow spoilers in practical systems only require simple flow parameters and corrected normalized spectrum $K^2(St)$ derived by Nelson and Morfey. It needs to be seen whether this predictive method based on their corrected normalized spectrum $K^2(St)$ can be applied to a wider range of flow duct discontinuities. A further study is required for the determination of values including coherence function in the interaction factor.

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